

A STRATEGY FOR OPTIMAL REPLACEMENT OF WATER PIPES INTEGRATING STRUCTURAL AND HYDRAULIC INDICATORS BASED ON A STATISTICAL WATER PIPE BREAK MODEL

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Abstract

The ageing of water networks results in an increase in water pipe breaks in addition to a decrease in hydraulic capacity. Considering the complexity of these processes combined with the huge investments municipalities will have to make to maintain an adequate service level, it is imperative to develop tools that will assist water supply utilities managers in selecting, among the available options, those that will minimize the total costs on the long term. This article presents a new strategy for the optimal replacement of water pipes. It integrates the two key elements involved in the deterioration of water supply services, namely the structural integrity and the hydraulic capacity. The objective function used to define optimal solutions comprises two terms: one related to repair costs and another to replacement costs. The optimal solution must minimize this function under the constraints that all node demands and pressure are satisfied. The model used to estimate the probability of pipe break occurrences considers time intervals between successive pipe breaks as a random variable described by probability density functions. A Bayesian approach is used to estimate the model parameters values. Network hydraulics are modeled using Epanet2.0, and a genetic algorithm (GA) is used to seek the optimal solution. The validation and the performance evaluation of the proposed strategy have been realized by generating stochastic pipe breaks on a water pipe network. The network "lifetime" has been subdivided into five-year time intervals. The planning schedule for the next five years is defined at the beginning of these time intervals (i.e. which pipes are replaced, and when they are replaced according to the optimization results). At the beginning of each of these periods, parameters values of the pipe break model are re-evaluated according to break records available at that time. Once the water pipe network is upgraded, pipe break records are extended to the next five years. This process of identifying water pipes to be replaced for the next five years is repeated until the end of the network "lifetime". Results are reported for two hypothetical water networks of 100 and 250 pipes, respectively.

Keywords

Pipe break model, cost, pressure, network, genetic algorithm, replacement, Bayesian inference.

1. INTRODUCTION

Distribution networks deteriorate over time. Pipe breaks frequency increases and the overall hydraulic performance decreases. Since financial resources of municipalities are limited, and considering the complexity of the processes involved as well as data scarcity, it is important to develop analysis tools to assist managers in their rehabilitation/replacement decision-making process.

An economic analysis of this problem was performed in pioneer works on the subject [15,17]. These approaches define, for a given pipe, an optimal replacement time based on the minimization of total costs which include replacement and maintenance costs related to pipe break repairs. Other works allowed the development of global strategies for the whole network including criteria related to the network structural quality and hydraulic performance. Among others, we can mention the work of Kim and Mays ([9]) who considered, in addition to repair and replacement costs, those associated to various rehabilitation options and also to pumping costs. Kleiner *et al.* [10,11] proposed a long-term planning approach that integrates rehabilitation options and replacement, and where the structural integrity and hydraulic capacity of each pipe are analyzed simultaneously. Dandy and Engelhardt [3] used genetic algorithms to determine an optimal pipe replacement strategy. This strategy was applied to the Adelaide network (Australia). Their model integrates a budgetary constraint as well as constraints on minimal pressures and maximum flow velocity. Lastly, Halhal *et al.* [8] proposed a multi-objective approach where the first objective is related to costs and the second to hydraulic performance, physical integrity, network flexibility and water quality.

Our work differs from previous published studies by three aspects: 1) a statistical model is used to estimate pipe break probabilities, 2) a Bayesian approach is used to estimate the statistical pipe break model (SPBM) parameter values and 3) a validation approach has been adopted where the optimal replacement strategy is defined at fixed time intervals (every five years in the present case) based on available data at that time.

2. PROPOSED OPTIMAL REPLACEMENT STRATEGY

The proposed optimal replacement strategy integrates structural and hydraulic aspects. The structural quality of the network is evaluated by considering the number of breaks reported over a given time period for a given pipe. An increase in the number of pipe breaks over a time period is thus indicative of a structural deterioration and results in an increase in maintenance costs. The hydraulic performance is related to the respect of minimal pressure constraints at every network node. The reduction in hydraulic performance indeed causes pressure reduction at network nodes resulting from higher pressure losses due to the accumulation of deposits at pipe surface [16]. The proposed strategy is based, on one hand, on the estimate of the average number of pipe breaks to occur on each pipe and, on the other hand, on the expected evolution of pressures at the network nodes over time. The only option considered for improving the structural state and the hydraulic performance is pipe replacement. The “new” water pipes are supposed to have exactly the same characteristics as those replaced. The strategy thus aims at establishing which water pipe must be replaced and when it should be replaced in order to minimize total costs (i.e. replacement plus maintenance costs) and in order to respect minimal pressure constraints at all nodes at every time step.

2.1 Statistical pipe break model and parameter values inference method

The proposed statistical pipe break model (SPBM) assumes that time periods between successive pipe breaks are random variables. Probability distributions are used to describe these variables and different distributions are associated with the elapsed time between successive pipe break occurrences. The estimation of the distribution parameters is carried out using available pipe break records. A classification of pipe breaks according to pipe material, periods of installation or to other variables likely to modify break probabilities can also be considered. This type of model was used by many authors to analyze pipe break records (e.g. [5,7,12]).

The SPBM considered here assumes that the time between installation and the first pipe break is described by a Weibull distribution whereas the time between successive pipe breaks (2nd and 3rd, 3rd and 4th, etc.) is represented by exponential distributions. Exponential distribution parameters, λ , (only one parameter is necessary to specify exponential distribution; this parameter is equal to the risk function [2]) are supposed to increase linearly with pipe break orders. The model can thus reproduce the observed increase in pipe break probabilities on a given pipe as the number of pipe breaks increases (see [12] for a detailed description of this model).

The SPBM parameter estimation is achieved using Bayesian inference (see [4,13] for details). This approach offers many advantages. First, it allows a simple integration of historical pipe break records as they become available and a simple update of the SPBM parameters. Secondly, it generates the SPBM parameter posterior distributions thus giving some uncertainty assessment of the parameter estimates. These uncertainties could be integrated into the analysis in order to see their impact on the decision-making process (Is the replacement schedule sensitive to the estimated SPBM parameter values?). Maximum values of posterior parameter distributions (mode) have been used as estimators of the SPBM parameter values. These parameter values are estimated separately for each pipe break order until the number of pipes having experienced a break of a given order is too small. This criterion aims at ensuring that the parameter estimates are statistically reliable. Parameter value estimates for higher pipe break orders, for which very little or no information is available, are carried out by assuming the following non-linear relation between parameter values and the pipe break order i :

$$\lambda(i) = a + b i^c \quad (1)$$

Parameters a , b , and c are estimated after fitting equation (1) to the low order exponential parameter values obtained from the Bayesian inference. Equation (1) thus allows the estimation of distribution parameter values for pipe breaks of higher order (see [4,13]).

2.2 Cost function and hydraulic constraint

The replacement strategy is formulated in terms of minimizing the cost function. This cost function includes pipe break repair and replacement costs. In addition, the required solutions must fulfill constraints of minimal pressure at network nodes. The cost function for the whole network is the sum of the cost function for each pipe. This cost is given by ([12]) (costs are discounted to year T ; installation time is set at $t = 0$ for all pipes):

$$C(j, Tf(j) | k(j)) = \frac{Cr(j) \ell(j)}{(1+R)^{[Tf(j)-T]}} + \sum_{i=1}^{k(j)} \frac{Cb(j)}{(1+R)^{[Ti(j)-T]}} + \sum_{t=T+1}^{Tf(j)} \frac{Cb(j)M(t-1, t | k(j), T)}{(1+R)^{(t-T)}} \quad (2)$$

where T is the time at which replacement schedule is elaborated; $Tf(j)$ is the replacement time of pipe j ; $k(j)$ is the number of pipe breaks sustained by pipe j during time period $[0, T]$; $M(t-1, t | k(j), T)$ is the estimated average number of pipe breaks to occur on pipe j during the year $(t-1)$ knowing that it has sustained $k(j)$ during period $[0, T]$; $Cr(j)$ is the linear replacement cost of pipe j (\$/m); $\ell(j)$ is the length of pipe j (m); R is the discount rate per year; $Ti(j)$ is the time of occurrence of the i^{th} pipe break on pipe j ; $Cb(j)$ is the average cost of repair (\$/pipe break). The first term on the right-hand side of equation (3) is the discounted replacement cost of pipe j at time Tf , the second is the discounted total repair cost of pipe j for period $[0, T]$, and the last term is the

estimated average repair cost associated to pipe breaks that will occur during period $[T+1, T_f]$ (from analysis to replacement time). The average number of pipe breaks to occur each year for the period $[T+1, T_f]$ is estimated using the SPBM and equations relating the SPBM parameters to the average number of pipe breaks [12]. The average number of pipe breaks is estimated using the parameter values of equation (1). The hydraulic constraint is written as:

$$Hc = \max_{\{t, t \in [T+1, T+\Delta t]\}} \left[\max_{\{m, m \in [1, M]\}} \left[\max_{\{H(m, t)\}} (0, H_{\min}(m, t) - H(m, t)) \right] \right] \quad (3)$$

where m is the node index; M is the number of nodes in the distribution network; $H(m, t)$ is the head at node m at time t and $H_{\min}(m, t)$ is the minimal pressure to maintain at node m at time t and Δt is the time period considered. The hydraulic penalty, Hc , is equal to the maximum pressure deficit to occur at all nodes during period $[T+1, T+\Delta t]$. Equation of Sharp and Walski [16] was considered to describe how Hazen-Williams coefficients evolved in time. The objective function is thus the sum of the total cost (sum of all pipe cost functions (eq. 2)) and the hydraulic penalty (eq. 3). The hydraulic penalty is multiplied by a weight (a constant) penalizing solutions that do not satisfy hydraulic constraint. The hydraulic simulator Epanet 2.0 [14] is used to carry out the hydraulic simulations.

The search for optimal solutions was performed using a genetic algorithm (GA). A generational GA using several demes (sub-populations) evolving separately was selected. The same set of operations is carried out in each deme at each generation, namely: tournament selection, one-point crossover and mutation, and, from time to time, migration between demes. When the maximum number of generations is reached, the individual (solution) with the best objective function is retained. This GA was implemented using the Open BEAGLE C++ framework [6].

2.3 Proposed replacement strategy and validation procedure

Validation of the proposed replacement strategy was carried out by subdividing the network “lifetime” in periods of equal duration called planning phases (PP) (Figure 1). The duration of the PP was set at five years. The planning of the replacement schedule (i.e. list of pipes to be replaced and years of replacement) to be carried out at each PP is done at the beginning of each of these PP (times T_1, T_2, T_3 , etc. of Figure 1). This analysis integrates all available information at that time (data on hydraulic parameters and pipe break records).

The suggested replacement strategy and its validation primarily comprise three steps. A first step aims at estimating the SPBM parameter values, on the basis of data and information available at the beginning of the PP. The second step consists in finding solutions that minimize the cost function and that satisfy hydraulic constraints. The list of pipes to be replaced over the forthcoming PP is determined. The third step consists in updating the network for the forthcoming phase. This means: 1) to carry out the scheduled pipe replacements at prescribed times, 2) to update pipe break records (stochastic generation of pipe breaks on all pipes for the forthcoming PP) and 3) to update Hazen-Williams coefficients. A method validation was performed using synthetic pipe break records, randomly generated by the SPBM. Once the network parameters and variables for the forthcoming PP have been updated, analysis for the next PP can be carried out and the replacement schedule for this new PP can be established. This validation procedure, though applied to hypothetical pipe break records, allows the evaluation of the long-term performance of the proposed strategy over sets of possible historical pipe break records and explicitly integrates available data.

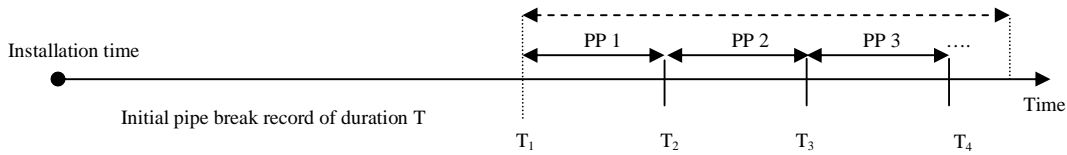


Figure 1. Proposed validation strategy

3 APPLICATION EXAMPLE

Two hypothetical networks were considered to validate the approach. The first one has 100 pipes, 60 nodes and 40 loops (Figure 2), and the second one has 250 pipes, 109 nodes and 109 loops. These networks are supplied by one source node with total heads of 80 m. All pipes are 100-meter long and possible diameters for the 100-pipe network are 100, 150, 250 or 500 mm (a detailed description of these networks is presented in [4]). Pipe break records are generated using the model described in section 2.1 with parameter values $a = -0.05$, $b = 0.05$ and $c = 1.0$ for equation 1 (linear increase of risk over time for break orders larger than one). A randomly generated record is considered in the following. The installation years of all pipes are arbitrarily set at $t = 0$. A 30-year pipe break record is initially generated for all pipes ($T = 30$ years). In the example considered here, pipes 16 and 28 have recorded the greatest number of breaks, which are 11 and 9 respectively (see Figure 2). For this application, the prior distributions for all SPBM parameters are Gamma distributions [4]. Replacement and repair costs are estimated using the equations proposed by [1].

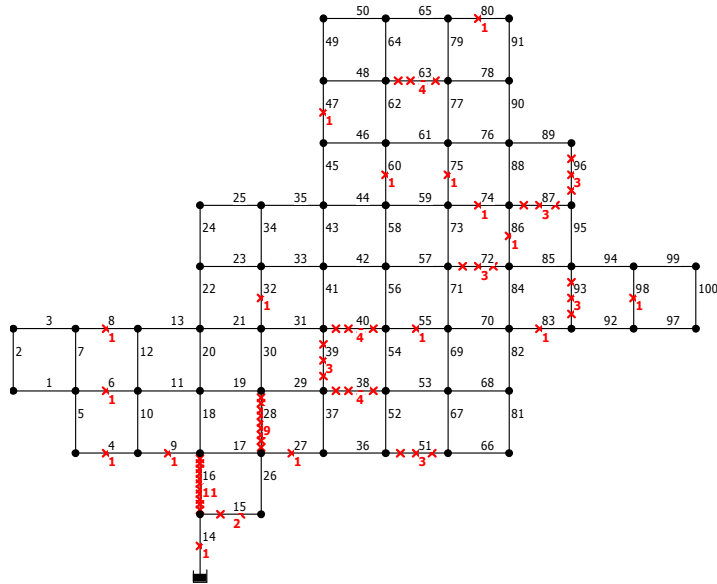


Figure 2 Example of an historical pipe break record for the 100-pipe network (“x” represents pipe breaks and numbers next to the “x” are the number of pipe breaks; numbers next to lines are the number of pipes)

4 RESULT AND DISCUSSION

Tests were initially done without the hydraulic constraint in order to validate the inference methodology of the SPBM parameters and to assess the performance of the GA. Table 1 presents an example of a replacement schedule obtained in that case for a random record. The number of pipes replaced, the corresponding time of replacement and the number of pipe breaks recorded prior to replacement are given. These results show that the first pipes replaced (pipes 16 and 28 at year 41 in our example) are those with the largest number of breaks (see Fig. 2). The fact that these pipes are replaced after 15 and 16 pipe breaks is related to the choice of prior distributions for the SPBM parameters. The actual choice is, in some sense, “conservative” and favours late replacement. As available pipe break records are longer, replacement is achieved after 8 to 10 pipe breaks (except for pipe 83 which will have recorded 18 pipe breaks before replacement; this is due to the fact that replacement schedules are planned at five-year intervals, and that during those five years, many pipe breaks can occur on a given pipe).

These results are in accordance with Mailhot *et al.*'s analysis which demonstrates that, in the context of a replacement strategy similar to the one considered here, where average breaks are estimated using a SPBM Weibull exponential type, replacement occurs when the pipes have reached a critical number of breaks. The actual number of replacements for a given PP depends on the estimated values of the SPBM, which is based on the available pipe break record. No budget constraint is imposed for replacement during a given PP.

The results also demonstrate, as expected, that uncertainties on SPBM estimated parameter values will be larger when pipe break records are shorter and/or the number of pipes in the network is smaller. These estimates improve when longer pipe break records are available or when networks with a larger number of pipes are considered. This also means that these estimates will be more sensitive to prior information if only a few data are available. This is particularly the case for high break orders since longer records are necessary to observe a statistically significant number of pipe breaks. Figure 3 shows an example where, for a given pipe break record, values of exponential distribution parameters obtained by the Bayesian inference for various pipe break orders are compared with the exact values used to generate the record. As expected, longer records improved the parameter estimates, especially for low break orders. These issues (i.e. impacts of the length of pipe break records, number of pipes and prior distributions on the estimators of SPBM parameter values) are discussed in [4,13].

When hydraulic constraints are included, the solutions must be simulated using the hydraulic simulator in order to estimate the corresponding maximum pressure deficit. Cost functions of solutions not complying with the hydraulic constraints are strongly penalized but these solutions are not automatically discarded. In order to understand the impact of adding a minimal pressure constraint on replacement, we used the previous example where a 17-m minimal pressure was first considered. The “best” solution obtained in this case for the first PP corresponds to the solution previously obtained when no hydraulic constraint is considered since all node pressures comply with this minimal pressure. If the minimal pressure is set at 20 m, node 58 (at the intersection of pipes 94 and 99) becomes critical since its estimated pressure is 19.2 m at the 30th year. Optimization results considering a 20-m minimal pressure show that the “best” solution corresponds to the replacement of only one pipe, that is pipe 94 at the 32nd year. This pipe is replaced for hydraulic reasons since it has not recorded any pipe break and its structural integrity is presumably good. No replacement of the pipes that have recorded breaks would solve the pressure problem at node 58.

Table 1. Example of an optimal replacement schedule (100-pipe network)

Planning phase	Number of replaced pipes	(Pipe number, years of replacement, number of pipe breaks at replacement time)
]30,35]	0	
]35,40]	0	
]40,45]	4	(16, 41,16) (28,41,15) (63,41,12) (96,44,9)
]45,50]	0	
]50,55]	8	(4,53,8) (8,51,9) (15,51,9) (27,55,7) (38,51,9) (39,51,13) (40,51,9) (87,51,10)
]55,60]	2	(51,56,10) (93,59,8)
]60,65]	2	(6,64,9) (14,64,9)
]65,70]	2	(70,68,11) (83,66,18)
]70,75]	8	(1,72,10) (9,74,9) (28,74,9) (30,74,9) (72,71,11) (73,74,9) (74,72,10) (85,74,9)
]75,80]	4	(24,76,10) (60,76,12) (75,80,8) (76,78,9)

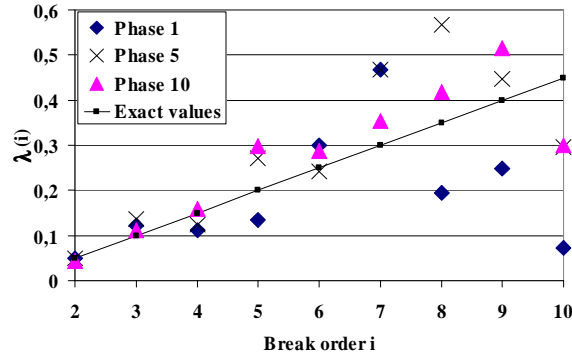


Figure 3 Exact and estimated values (maximum value of the posterior distribution) of the SPBM exponential distributions for various phases and various pipe break orders (100-pipe network).

This example is typical and illustrates some drawbacks to using a mono-objective optimization scheme to consider hydraulic constraints in the replacement decision-making process. It shows that hydraulic constraints can lead to pipe replacements even though this doesn't improve the overall structural state. As previously stated, the optimization objective is to search first among the "structural" solutions (pipe replacements obtained by minimizing total costs), the one that will fulfill hydraulic requirements. If none of these solutions satisfies this criterion, pipes will be selected for replacement, which will lead to a network with node pressures above the prescribed minimal pressure. Hydraulic improvement cannot be achieved through structural improvement in such situations. Hydraulic constraints can, of course, be relaxed in order to favour solutions that will simultaneously increase the network hydraulic performance and the structural integrity. However, this implies, in the context of a mono-objective, that arbitrarily defined weighing factors should be sought for among hydraulic and structural terms of the objective function.

5 CONCLUSION

The deterioration of the structural state of water pipes in distribution networks represents a real problem for the network managers considering the difficulty to assess the structural state and to link this type of information to that related to hydraulic performance. The proposed replacement strategy integrates both structural and hydraulic aspects. The evaluation of the structural state is carried out by using a statistical model that estimates pipe break occurrence probabilities. The parameters of this model are estimated by a Bayesian inference using available pipe break records. This approach allows a simple update of parameter values [4,13]. The parameter value posterior distributions can also be used to estimate how uncertainties on these values will modify the replacement strategy (the estimator used here is the posterior distribution maximum value). Parameter value estimates of high break orders are carried out considering that exponential distribution parameters are non-linearly related to break orders. The hydraulic performance is evaluated by verifying that the minimal pressure constraint is fulfilled at anytime at all network nodes.

The optimal replacement strategy (identification of the pipes to be replaced with replacement years) is achieved through minimization of the total cost function (replacement and repair costs). Solutions must also comply with the minimal pressure constraint. The hydraulic constraint is considered by adding a penalty to the cost function of solutions that do not satisfy minimal pressure constraints. A genetic algorithm (GA) is used to find the optimal solutions. The validation of this approach was carried out by subdividing the "lifespan" of the network in phases of equal durations (five years in the present case). The optimal replacement schedule is defined at the beginning of each of these periods and aims at establishing the list of pipes to be replaced and their replacement time for the forthcoming five-year period. Once defined, the replacements are done at prescribed times. The network is then updated (pipe break records and hydraulic parameters) for the beginning of the next planning period. Pipe break records are generated using the SPBM while Hazen-Williams parameters are estimated using

Sharp and Walski equation [16]. This validation procedure emulates the operational context and makes possible an evaluation of the long-term performance of the proposed replacement strategy.

A first application of this approach was done on two hypothetical networks of 100 and 250 pipes respectively. Pipe break records were generated randomly using the SPBM. The results show that the accuracy of the estimated SPBM parameter values depends, as expected, on the length of the available pipe break record. Uncertainties on higher order parameters are larger because of the small number of pipe breaks available. The choice of prior distributions must be carefully examined if pipe break records are short [4,13]. This approach has the advantage of updating the SPBM parameters as pipe break data become available. The hydraulic constraint was incorporated in the optimization process by penalizing solutions that do not comply with the prescribed minimal pressure. This favours, among the solutions improving the structural integrity, those that will satisfy hydraulic constraints. However, examples showed that this will sometimes lead to pipe replacements exclusively justified for hydraulic reasons even though no break occurred. A complete analysis is underway to estimate how often this occurs for randomly generated pipe break records.

Obviously, such a replacement strategy can hardly be justified from an operational standpoint. On the other hand, the replacement strategy based on the structural integrity could lead to situations where the network hydraulic performance becomes unacceptable. In such a context, it is necessary to define an equilibrium point where optimal solutions are achieved for both hydraulic performance and structural integrity objectives. Within a mono-objective optimization framework, the identification of the relative weight of these objectives is required in the objective function, which can be a cumbersome and somehow arbitrary exercise.

The use of a multi-objective approach seems interesting for several reasons. It would indeed allow the search for sets of solutions allowing simultaneous optimization of the structural integrity and the hydraulic performance objectives. Moreover, such an approach gives the manager the opportunity to choose, among a set of solutions, the one that could satisfy some other criteria not taken into account in the optimization process. Works are currently underway to implement a multi-objective approach.

ACKNOWLEDGEMENTS

The authors would like to thank Joëlle Bérubé and Diane Tremblay of INRS-ETE for their excellent proofreading.

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